МАТЕМАТИКА

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On the boundary conditions of the wave potential in a domain with a curviline borders

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Dedicated to the memory of our Dear Teacher
Nakhushev Adam Maremovich

1. Introduction. Volume potentials for partial equations due to their theoretical and applied significance are one of important notions of the modern theory of differential equations. Key stages of this theory are the researches held by Newton for elliptic potentials, where a practical significance of the problem was also shown alongside with fundamental researches of a whole row of essential questions of this theory. Different applications of the volume potential in electrostatics, heat-conductivity, elasticity, diffusion and other fields of the science are well-known and were of interest to such scientists as Laplace, Gauss, Poisson, Green, Beltrami, Kirchhoff, lord Kelvin, Hobson, Lyapunov, Sobolev, Bitsadze and others, who made a great contribution to the development of this theory during several centuries.

The volume elliptic potential is widely used in solving classical problems of Dirichlet, Neumann and other boundary value problems for domains of an arbitrary form. But, at the same time, boundary conditions and spectral problems of the volume potential have not been researched till recent time. That is, despite the deep research of the general theory of the volume potential, till the recent time the Newton volume potential

$$u_{NP}(x) = \int_{\Omega} \varepsilon(x-y) f(y) dy$$

has not been considered as an independent operator being a solution of some boundary value problem.

The works of T.Sh. Kal'menov, his disciples and followers [1–9] laid the foundations of the theory of boundary value problems for different kinds of the volume potentials. And in the world literature such scientists as Engquist B. and Majda A. [10], Givoli D. [11]-[13], Li J.R., Greengard L. [14], Hagstrom T. [15], Tsynkov S.V. [16], Saito N. [17], Wu X. and Zhang J. [18] also used analogous research results for solving various problems of the mathematical physics and numerical calculations.

New non-local boundary conditions which uniquely define the Newton volume potential, have the form

$$\frac{u(x)}{2} - \int_{\partial\Omega} \left(\frac{\partial \varepsilon(x-y)}{\partial n_y} u(y) - \varepsilon(x-y) \frac{\partial u(y)}{\partial n_y} \right) dS_y = 0, \quad x \in \partial\Omega.$$

In particular in the papers [1,2], by using a new non-local boundary value problem, which is equivalent to the Newton potential, the authors founded explicitly all eigenvalues and eigenfunctions of the Newton potential in the 2-disk and the 3-ball.

The aim of this paper is to give an analogy of the boundary value problem for the wave potential. Unlike elliptic and parabolic cases, where the obtained boundary conditions for corresponding volume potentials are non-local , for the wave potential we get a local initial boundary value condition. Note that the case of the volume wave potential in the domain with rectilinear boundaries was considered in [6].

2. Formulation of problem. We consider the following wave potential

$$u(x,t) = \iint_{\Omega} \varepsilon(x-\xi, t-\tau) f(\xi,\tau) d\xi d\tau, \tag{1}$$

where $\varepsilon(x,t) = \frac{1}{2}\theta(t-|x|)$ is a fundamental solution of Cauchy problem for the wave equation:

$$\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t), (x,t) \in \Omega, \tag{2}$$

with the initial conditions

$$u(x,0) = u_t(x,0) = 0, 0 < x < 1. \tag{3}$$

Here $\Omega \subset \mathbb{R}^2$ is a finite domain bounded at the sides by the curves $x = \alpha(t)$ and $x = \beta(t)$, and bounded above and below by the segments $t = 0$, $0 < x < 1$ and $t = T$, $x_0 < x < x_1$. Here $T > 0$, $\alpha(0) = 0$, $\beta(0) = 1$, $\alpha(T) = x_0$, $\beta(T) = x_1$, $\alpha(t) < \beta(t)$. Additionally assume that

$$|\alpha'(t)| < 1, \quad |\beta'(t)| < 1. \tag{4}$$

It is known that for $T > 1/2$ the solution of the wave equation (2) in Ω is restored under the initial conditions (3) not uniquely. For the uniqueness it is necessary to use boundary conditions. We set a task to construct boundary conditions under which (together with the initial conditions (3)) the solution of Eq. (2) in Ω will be uniquely defined in the form (1). In the case when $\alpha(t) \equiv 0$ and $\beta(t) \equiv 1$, this problem was considered in [6].

3. Construction of boundary conditions. By $\Omega_{x,t}$ we denote a part of Ω: $\Omega_{x,t} = \{(\xi,\tau) \in \Omega : |x-\xi| < t-\tau|\}$. Then the volume potential (1) can be written in the form

$$u(x,t) = \frac{1}{2} \iint_{\Omega_{x,t}} f(\xi,\tau) d\xi d\tau, \tag{5}$$

Evidently that for any $f(x,t) \in C^1(\overline{\Omega})$ the volume potential (5) gives a classical solution of the inhomogeneous wave equation (2) from the class $u(x,t) \in C^2(\overline{\Omega})$. Our task is to construct homogeneous boundary conditions to which the volume potential (5) satisfies for all $f(x,t)$.

We consider separately various cases of placing $\Omega_{x,t}$ inside Ω.

3.1 Case I

Firstly we consider a case when $0 \le x-t < x+t \le 1$. In this case the domain $\Omega_{x,t}$ nowhere touches the side boundaries Ω (see Figure 1).

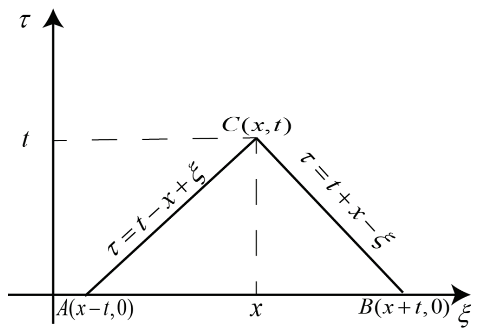

Figure 1 – the domain $\Omega_{x,t}$ in the case I

Therefore there is no need to construct the boundary conditions for the wave potential. By direct calculation it is easy to see that the volume potential (5) satisfies the homogeneous initial conditions (3).

3.2 Case II

Now let $x = \alpha(t)$ and $t + \alpha(t) \leq 1$. By virtue of the condition (4) it is easy to see that $t + \alpha(t) > 0$ in Ω. In this case the domain $\Omega_{x,t}$ coincides with curvilinear triangle, in the foundation of which there lays a segment $0 < \xi < t + \alpha(t)$ of the axis $\tau = 0$. The left-hand side of the triangle is a curvilinear segment $\xi = \alpha(\tau)$ for $0 < \tau < t$. The right-hand side of the triangle is a segment of the straight line $\xi = t + \alpha(t) - \tau$ for $0 < \tau < t$ (see Figure 2).

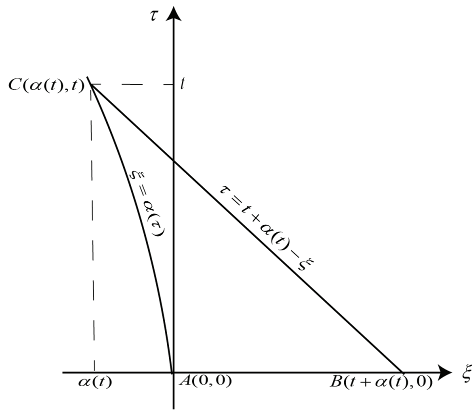

Figure 2 – the domain $\Omega_{x,t}$ in the case II

Hereinafter we will use the Green's theorem [19]: *Let C be a positively oriented, piecewise smooth, simple closed curve in a plane, and let D be a domain bounded by C. If L and M are functions of (ξ, τ) defined on an open domain containing D and have continuous partial derivatives there, then*

$$\oint_C (L\, d\xi + M\, d\tau) = \iint_D \left(\frac{\partial M}{\partial \xi} - \frac{\partial L}{\partial \tau} \right) d\xi\, d\tau,$$

where the left-hand side is a line integral and the right-hand side is a surface integral, and the path of integration along C is anticlockwise.

Applying Green's theorem, we get the following chain of equalities:

$$u\left(\alpha(t), t\right) = \frac{1}{2} \iint_{\Omega_{\alpha(t),t}} f(\xi,\tau) d\xi d\tau = \frac{1}{2} \iint_{\Omega_{\alpha(t),t}} \left(\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} - \frac{\partial^2 u(\xi,\tau)}{\partial \xi^2} \right) d\xi d\tau$$

$$= -\frac{1}{2} \oint_{\partial\Omega_{\alpha(t),t}} \left(\frac{\partial u(\xi,\tau)}{\partial \tau} d\xi + \frac{\partial u(\xi,\tau)}{\partial \xi} d\tau \right).$$

Calculating the obtained line integrals, taking into account the initial conditions (3), we have

$$u\left(\alpha(t), t\right) = \frac{1}{2} \int_0^t \left[\frac{\partial u}{\partial \tau}(\alpha(\tau),\tau)\alpha'(\tau) + \frac{\partial u}{\partial \xi}(\alpha(\tau),\tau) \right] d\xi + \frac{1}{2} u\left(\alpha(t), t\right).$$

Hence

$$u\left(\alpha(t), t\right) = \int_0^t \left[\frac{\partial u}{\partial \tau}(\alpha(\tau),\tau)\alpha'(\tau) + \frac{\partial u}{\partial \xi}(\alpha(\tau),\tau) \right] d\tau. \tag{6}$$

Differentiating (6), we obtain

$$\left[\frac{\partial u}{\partial x}(\alpha(t),t) - \frac{\partial u}{\partial t}(\alpha(t),t)\right][1-\alpha'(t)] = 0.$$

Therefore, taking into account the conditions (4), we will have the boundary condition on a part of the left boundary

$$\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t}\right)(\alpha(t),t) = 0, \quad \text{at} \quad t+\alpha(t) \le 1. \tag{7}$$

3.3 Case III

Consider a case when $x = \beta(t)$ and $\beta(t) - t \ge 0$. By virtue of the condition (4) it is easy to see that $\beta(t) - t < 1$ in Ω. In this case the domain $\Omega_{x,t}$ coincides with curvilinear triangle, in the foundation of which there lays a segment $\beta(t) - t < \xi < 1$ of the axis $\tau = 0$. The right-hand side of the triangle is a curvilinear segment $\xi = \beta(\tau)$ for $0 < \tau < t$. And the left-hand side of the triangle is a segment of the straight line $\xi = \tau - t + \beta(t)$ for $0 < \tau < t$ (see Figure 3).

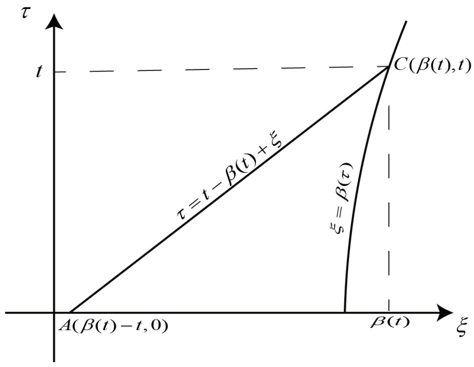

Figure 3 – the domain $\Omega_{x,t}$ in the case III

Analogously, as in Sec. 3.2, applying the Green's theorem, we obtain the boundary condition on a part of the right boundary

$$\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}\right)(\beta(t),t) = 0, \quad \text{at} \quad \beta(t) - t \ge 0. \tag{8}$$

3.4 Case IV

To consider this case is necessary if the domain Ω contains such points (x,t), for which $t + \alpha(t) > 1$ or $\beta(t) - t < 0$. Consider the domain $\Omega_{x,t}$ being the curvilinear pentagon, in the foundation of which there lays a segment $0 < \xi < 1$ of the axis $\tau = 0$. The left-hand side of the pentagon is a curvilinear segment $\xi = \alpha(\tau)$, and the right-hand side is a curvilinear segment $\xi = \beta(\tau)$. The pentagon is bounded above by the segments of the straight line $\tau - \xi = t - x$ and $\tau + \xi = t + x$ (see Figure 4).

We apply the Green's theorem for the volume wave potential

$$u(x,t) = \frac{1}{2}\iint_{\Omega_{x,t}} f(\xi,\tau)d\xi d\tau = \frac{1}{2}\iint_{\Omega_{x,t}} \left(\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} - \frac{\partial^2 u(\xi,\tau)}{\partial \xi^2}\right) d\xi d\tau.$$

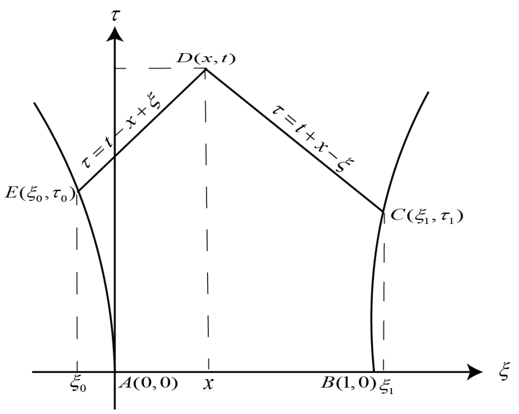

Figure 4 – the domain $\Omega_{x,t}$ in the case IV

Then we obtain the identity

$$\begin{aligned} &u\left(\alpha(\tau_0),\tau_0\right)+u\left(\beta(\tau_1),\tau_1\right)-\int_0^{\tau_0}\left[\frac{\partial u}{\partial\tau}(\alpha(\tau),\tau)\alpha'(\tau)+\frac{\partial u}{\partial\xi}(\alpha(\tau),\tau)\right]d\tau \\ &\qquad+\int_0^{\tau_1}\left[\frac{\partial u}{\partial\tau}(\beta(\tau),\tau)\beta'(\tau)+\frac{\partial u}{\partial\xi}(\beta(\tau),\tau)\right]d\tau=0, \end{aligned} \tag{9}$$

where $(\alpha(\tau_0),\tau_0)$ is a point of crossing of the boundary curve $\xi=\alpha(\tau)$ and of the characteristics $\xi=\tau-t+x$; $(\beta(\tau_1),\tau_1)$ is a point of crossing of the boundary curve $\xi=\beta(\tau)$ and of the characteristics $\xi=t+x-\tau$. The existence of such points is provided by the fulfillment of the condition (4).

In (9) equating firstly $x=\alpha(t)$, and then $x=\beta(t)$, we get two identities

$$\begin{aligned} &u\left(\alpha(t),t\right)+u\left(\beta(\tau_1(t)),\tau_1(t)\right)-\int_0^{t}\left[\frac{\partial u}{\partial\tau}(\alpha(\tau),\tau)\alpha'(\tau)+\frac{\partial u}{\partial\xi}(\alpha(\tau),\tau)\right]d\tau \\ &\qquad+\int_0^{\tau_1(t)}\left[\frac{\partial u}{\partial\tau}(\beta(\tau),\tau)\beta'(\tau)+\frac{\partial u}{\partial\xi}(\beta(\tau),\tau)\right]d\tau=0, \end{aligned} \tag{10}$$

$$\begin{aligned} &u\left(\alpha(\tau_0(t)),\tau_0(t)\right)+u\left(\beta(t),t\right)-\int_0^{\tau_0(t)}\left[\frac{\partial u}{\partial\tau}(\alpha(\tau),\tau)\alpha'(\tau)+\frac{\partial u}{\partial\xi}(\alpha(\tau),\tau)\right]d\tau \\ &\qquad+\int_0^{t}\left[\frac{\partial u}{\partial\tau}(\beta(\tau),\tau)\beta'(\tau)+\frac{\partial u}{\partial\xi}(\beta(\tau),\tau)\right]d\tau=0, \end{aligned} \tag{11}$$

where $(\alpha(\tau_0(t)),\tau_0(t))$ is a point of crossing of the boundary curve $\xi=\alpha(\tau)$ and of the characteristics $\xi=\tau-t+\beta(t)$; $(\beta(\tau_1(t)),\tau_1(t))$ is a point of crossing of the boundary curve $\xi=\beta(\tau)$ and of the characteristics $\xi=t+\alpha(t)-\tau$.

By direct calculation, taking into account the conditions (4), it is easy to see that

$$\tau_0'(t)=\frac{1-\beta'(t)}{1-\alpha'(\tau_0(t))}\quad\text{and}\quad\tau_1'(t)=\frac{1+\alpha'(t)}{1+\beta'(\tau_1(t))}. \tag{12}$$

After differentiating equations (10) and (11) with respect to the variable t, using (12) we obtain

$$\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}\right)(\alpha(t),t)=\frac{1+\alpha'(t)}{1-\alpha'(t)}\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}\right)(\beta(\tau_1(t)),\tau_1(t)), \tag{13}$$

$$\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}\right)(\beta(t),t)=\frac{1-\beta'(t)}{1+\beta'(t)}\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}\right)(\alpha(\tau_0(t)),\tau_0(t))\,. \tag{14}$$

The identity (13) holds for $t+\alpha(t)>1$, and the identity (14) holds for $\beta(t)-t\ <0$.

Both obtained identities connect with each other traces of variables on the left-hand and right-hand boundaries of the domain. Herewith, since $t>\tau_1(t)$ and $t>\tau_0(t)$, then the points in which values are taken in the left-hand parts of this identities, are "above" than the points, in which values are taken in the right-hand parts of the identities. Therefore, taking into account the boundary conditions (7) and (8) calculated in Case II and Case III, from (13) and (14) we get the proof of the following lemma.

Lemma 1. *The volume wave potential* (1) *satisfies the wave equation* (2)*, the homogeneous initial conditions* (3)*, the boundary condition on the left-hand boundary of the domain*

$$\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial t}\right)(\alpha(t),t)=0,\quad at\ \ 0\le t\le T, \tag{15}$$

and the boundary condition on the right-hand boundary of the domain

$$\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}\right)(\beta(t),t)=0,\quad at\ \ 0\le t\le T. \tag{16}$$

Corollary 1. *The volume wave potential* (1) *is the solution of the initial-boundary value problem* (2)*,* (3)*,* (15)*,* (16)*.*

4. Uniqueness of solution of problem (2)**,** (3)**,** (15)**,** (16) The constructed in Section 3 boundary conditions (15), (16) will uniquely define the volume wave potential (1), if the initial-boundary value problem (2), (3), (15), (16) has no other solutions except (1).

Lemma 2. *The solution of the initial-boundary value problem* (2)*,* (3)*,* (15)*,* (16) *is unique.*
Proof. As usual, by $u_1(x,t)$ and $u_2(x,t)$ we denote two solutions of the initial-boundary value problem (2), (3), (15), (16). Then their difference $u(x,t)=u_1(x,t)-u_2(x,t)$ satisfies the homogeneous wave equation

$$\frac{\partial^2 u(x,t)}{\partial t^2}-\frac{\partial^2 u(x,t)}{\partial x^2}=0,(x,t)\in\Omega, \tag{17}$$

the homogeneous initial conditions (3) and the homogeneous boundary conditions (15), (16).

Generally speaking, the proof must be held separately for cases Case II – Case III, as in Section 3. Here we consider only Case III. The rest cases are considered analogously.

We apply the Green's theorem to the integral

$$0=\iint_{\Omega_{x,t}}\left(\frac{\partial^2 u(\xi,\tau)}{\partial\tau^2}-\frac{\partial^2 u(\xi,\tau)}{\partial\xi^2}\right)d\xi d\tau.$$

Then we obtain the identity

$$\begin{gathered}u\left(\alpha(\tau_0),\tau_0\right)+u\left(\beta(\tau_1),\tau_1\right)-\int_0^{\tau_0}\left[\frac{\partial u}{\partial\tau}(\alpha(\tau),\tau)\alpha'(\tau)+\frac{\partial u}{\partial\xi}(\alpha(\tau),\tau)\right]d\tau\\+\int_0^{\tau_1}\left[\frac{\partial u}{\partial\tau}(\beta(\tau),\tau)\beta'(\tau)+\frac{\partial u}{\partial\xi}(\beta(\tau),\tau)\right]d\tau-2u(x,t)=0,\end{gathered} \tag{18}$$

where $(\alpha(\tau_0),\tau_0)$ and $(\beta(\tau_1),\tau_1)$ are defined in (9).

In (18) equating firstly $x = \alpha(t)$, and then $x = \beta(t)$, we get two identities. After differentiating these identities with respect to the variable t, we obtain

$$(u_x - u_t)(\alpha(t), t)(\alpha'(t) - 1) + (u_x + u_t)(\beta(\tau_1(t)), \tau_1(t))(\alpha'(t) + 1) = 2\frac{d}{dt}u(\alpha(t), t), \tag{19}$$

$$(u_x + u_t)(\beta(t), t)(1 + \beta'(t) - (u_x - u_t)(\alpha(\tau_0(t)), \tau_0(t))(1 - \beta'(t) = 2\frac{d}{dt}u(\beta(t), t)). \tag{20}$$

Taking into account the homogeneous boundary conditions (15), (16), from (19), (20) we obtain that $u(\alpha(t), t) = Const$ and $u(\beta(t), t) = Const$ for all values $0 \leq t \leq T$. Taking into account the homogeneous initial conditions (3), from here we find

$$u(\alpha(t), t) = u(\beta(t), t) = 0, \quad \text{at} \quad 0 \leq t \leq T. \tag{21}$$

Thus, the function $u(x, t)$ satisfies the homogeneous wave equation (17), the homogeneous initial conditions (3) and the homogeneous boundary conditions (21), that is, it is the solution of the homogeneous first initial-boundary value problem. By virtue of the uniqueness of its solution we have $u(x, t) \equiv 0$ at $(x, t) \in \Omega$. Consequently, $u_1(x, t) \equiv u_2(x, t)$. The Lemma is proved.

5. Formulation of main result

Определение 1. *By the classical solution of the initial-boundary value problem* (2), (3), (15), (16) *we will call a function* $u(x, t)$ *from the class* $u(x, t) \in C^2\left(\overline{\Omega}\right)$ *satisfying Eq.* (2) *and the conditions* (3), (15), (16).

Combining the results of Lemma1 and Lemma 2, we get the main result of the paper.

Theorem 1. *Let* $f(x, t) \in C^1\left(\overline{\Omega}\right)$. *The volume wave potential* (1) *satisfies the wave equation* (2), *the homogeneous initial conditions* (3), *the boundary condition* (15) *on the left-hand boundary of the domain and the boundary condition* (16) *on the right-hand boundary of the domain.*
Inversely, for any $f(x, t) \in C^1\left(\overline{\Omega}\right)$ *the initial-boundary value problem* (2), (3), (15), (16) *has the unique classical solution* $u(x, t) \in C^2\left(\overline{\Omega}\right)$ *and this solution is represented in the form of the wave potential* (1).

Corollary 2. *The boundary conditions* (15), (16) *together with the initial conditions* (3) *uniquely define the volume wave potential* (1), *that is, they are the boundary conditions of the wave potential* (1).

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ABSTRACT

We study a one-dimensional volume wave potential in a domain with curvilinear boundaries. As a kernel of the wave potential we have chosen the fundamental solution of the Cauchy problem. It is well-known that in this case the volume wave potential satisfies one-dimensional initial conditions of Cauchy. We have constructed boundary conditions to which the wave potential satisfies at lateral boundaries of the domain. It is shown that the formulated initial-boundary value problem has the unique classical solution.

Keywords: wave equation, initial-boundary value problem, equation hyperbolic type, boundary condition, wave potential.

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АННОТАЦИЯ

В работе мы рассматриваем одномерный объемный волновой потенциал в области с криволинейными границами. В качестве ядра волнового потенциала выбрано фундаментальное решение задачи Коши. Хорошо известно, что в этом случае объемный волновой потенциал удовлетворяет однородным начальным условиям Коши. Мы построили краевые условия, которым удовлетворяет волновой потенциал на боковых границах области. Показано, что сформулированная начально-краевая задача имеет единственное классическое решение.

Ключевые слова: Иволновое уравнение, начально-краевая задача, уравнение гиперболического типа, граничное условие, волновой потенциал.

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