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Original article

On Bergman type projections in bounded strongly pseudoconvex domains

Romi F. Shamoyan, Elena B. Tomashevskaya

Bryansk State Technical University, Bryansk, Russia

rshamoyan@gmail.com, Tomele@mail.ru

Abstract. In our note we prove the boundedness of Bergman type projections in two different spaces of analytic functions with mixed norm in general bounded strongly pseudoconvex domains with smooth boundary. The first class of analytic functions was studied previously by many authors, the second function space however is completely new. Our proofs are based on standard known estimates of function space theory in bounded strongly pseudoconvex domains with smooth boundary and on some known estimates of Bergman kernel in such type domains. These estimates are also well-known in the unit disk. This allows us to provide proofs first in one dimensional case which are simpler and then repeating same arguments to show same type results also in more general situation. Note that many results on boundedness of Bergman type projections are well known and they have also various nice applications in complex function theory in one or several complex variable. Our results may also have various applications in complex function theory in bounded strongly pseudoconvex domains with smooth boundary.

Keywords: pseudoconvex domains, unit disk, analytic function, Bergman projection.

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Научная статья

О проекторах типа Бергмана в строго псевдовыпуклых ограниченных областях

Р. Ф. Шамоян, Е. Б. Томашевская

Брянский государственный технический университет, г. Брянск, Россия

rshamoyan@gmail.com, Tomele@mail.ru



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Аннотация. В данной работе нами доказывается ограниченность проекторов типа Бергмана в двух (разных) аналитических пространствах со смешанной нормой в общих ограниченных строго псевдовыпуклых областях с гладкой границей. Первый класс аналитических пространств уже изучался ранее различными авторами, второй же класс аналитических пространств является новым. В доказательствах нами используются некоторые уже известные оценки, полученные ранее в строго псевдовыпуклых ограниченных областях и стандартные оценки ядра Бергмана в ограниченных строго псевдовыпуклых областях с гладкой границей, что позволяет в некоторых случаях свести эти доказательства наших общих результатов к частному случаю единичного круга. Хорошо известно, что ограниченность тех или иных проекторов типа Бергмана в тех или иных классах аналитических функций имеет множество приложений в теории аналитических функций как в случае одномерном так и в многомерном случае. Результаты нашей заметки могут иметь различные приложения в теории пространств голоморфных функций в ограниченных строго псевдовыпуклых областях с гладкой границей.

Ключевые слова: аналитическая функция, проекторы типа Бергмана, единичный круг, псевдовыпуклые области.

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Introduction

We provide an extension of a classical result to pseudoconvex domains. Throughout this paper $H(D)$ or $H(\Lambda)$ denotes the space of all holomorphic functions on an open set $D \subset \mathbb{C}^n$ or $\Lambda \subset \mathbb{C}^n$. We denote by dv the normalized Lebesgue measure on D .

We follow notation from [1],[2],[3]. Let D be a bounded strongly pseudoconvex domain in \mathbb{C}^n with smooth boundary, let $d(z) = \text{dist}(z, \partial D)$. Then there is a neighbourhood U of \bar{D} and $\rho \in C^\infty(U)$ such that $D = \{z \in U : \rho(z) > 0\}$, $|\Delta \rho(z)| \geq c > 0$ for $z \in \partial D$, $0 < \rho(z) < 1$ for $z \in D$ and $-\rho$ is strictly plurisubharmonic in a neighborhood U_0 of ∂D . Note that $d(z) \asymp \rho(z)$, $z \in D$.

Then there is an r_0 such that the domains $D_r = \{z \in D : \rho(z) > r\}$ are also smoothly bounded strictly pseudoconvex domains for all $0 < r \geq r_0$. Let $d\sigma_r$ be the normalized surface measure on C^n . The following analytic mixed norm spaces were investigated in [1],[2],[3].

For $0 < p < \infty$, $0 < q \leq \infty$, $\delta > 0$, $k = 0, 1, r$ we set

$$\|f\|_{p,q,\delta,k} = \sum_{|\alpha| \leq k} \left(\int_0^{r_0} \left(\int_{\partial D_r} |D^\alpha f|^p d\sigma_r \right)^{q/p} \frac{dr}{r} \right)^{1/q}, \quad 0 < q < \infty;$$

$$\|f\|_{p,\infty,\delta,k} = \sup_{0 < r < r_0} \sum_{|\alpha| \leq k} \left(r^\delta \int_{\partial D_r} |D^\alpha f|^p d\sigma_r \right)^{1/p}.$$

The corresponding $A_{\delta,k}^{p,q} = \{f \in H(D) : \|f\|_{p,q,\delta,k} < \infty\}$ are complete quasinormed spaces for $p, q \geq 1$ They are Banach spaces. We mostly deal with the case $k = 0$ then we write simply $A_\delta^{p,q}$ and $\|f\|_{p,q,\delta}$.

We also consider these spaces for $p = \infty$ and $k = 0$, the corresponding space is denote by $A_\delta^{\infty,p} = A_\delta^{\infty,p}(D)$ and consists of all $f \in H(D)$ such that

$$\|f\|_{\infty,p,\delta} = \left(\int_0^{r_0} \left(\sup_{\partial D_r} |f| \right)^p \cdot r^{\delta p - 1} dr \right)^{1/p} < \infty.$$

Also for $\delta > -1$ the space $A_\delta^\infty = A_\delta^\infty(D)$ consists of all $f \in H(D)$ such that

$$\|f\|_{A_\delta^\infty} = \left(\sup_{z \in D} |f(z)| \rho(z)^\delta < \infty.\right)$$

and the weighted Bergman space A_δ^p .

$A_\delta^p = A_{\delta+1}^{p,p}(D)$ consists of all $f \in H(D)$ such that

$$\|f\|_{A_\delta^p} = \left(\int_D |f(z)|^p \rho(z)^\delta dv(z) \right)^{1/p} < \infty,$$

where dv is a normalized Lebegues measure in D .

Let further $dv_\alpha = (\delta^\alpha)dv(z)$ where $\delta(z) = \rho(z)$. Our proofs are heavily based on the estimates from [4] where more general situation was considered. Since $|f(x)|^p$ is subharmonic (even plurisubharmonic) for a holomorphic f we have $A_s^p(D) \subset A_t^\infty(D)$, for $0 < p < \infty, sp > n$ and $t = s$, also $A_s^p(D) \subset A_s^1(D)$ for $0 < p \leq 1$ and $A_s^p(D) \subset A_t^1(D)$ for $p > 1$ and t sufficiently large.

Therefore we have an integral representation

$$f(z) = \int_D f(\xi) K_{\tilde{t}}(z, \xi) \rho^t(\xi) dv(\xi), \tag{1}$$

$f \in A_t^1(D), z \in D, \tilde{t} = t + n + 1$, where $K_{\tilde{t}}(z, \xi)$ is a kernel of type t that is a smooth function on $D \times D$ such that

$$|K_{\tilde{t}}(z, \xi)| \leq C_1 |\tilde{\Phi}(z, \xi)|^{-(n+1+t)},$$

where $\tilde{\Phi}(z, \xi)$ is so called Henkin-Ramirez function on D . Note that (1) holds for function in any space X that embeds into A_t^1 . We review some known facts on $\tilde{\Phi}$. This function is C^∞ in $U \times U$ where U is a neighborhood of \bar{D} it is holomorphic in z and $\tilde{\Phi}(\xi, \xi) = \rho(\xi)$ for $\xi \in U$. Moreover on $\bar{D} \times \bar{D}$ it vanished only on the diagonal $(\xi, \xi), \xi \in \partial D$.

Locally it is up to a non vanishing smooth multiplicative factor equal to the Levi polynomial of ρ .

We denote various positive constants in this paper by C, C_1, C_2 ect.

Lemma 1. (See [1],[4]-[6].)

Assume $K_{\tilde{t}}(z, \xi)$ is a kernel of type $t; t > -1, \tilde{t} = t + n + 1$

a) For $0 < r < r_0$ we have

$$\int_{\partial D_r} |K_{\tilde{t}}(z, \xi)| d\sigma_r(z) \leq C_2(\rho(\xi) + r)^{-t-1}; \xi \in D;$$

b) Assume $\sigma > 0$ satisfies $\sigma - t - 1 < 0$. Then we have

$$\int_D |K_{\tilde{t}}(z, \xi)| \rho^{\sigma-1}(z) dv(z) \leq C_3 \rho^{\sigma-t-1}(\xi); \xi \in D.$$

Note that the same estimates are valid if K is replaced by $\tilde{K}_t(z, \xi) = K_t(\xi, z), t > 0$. Some facts on $A_{\alpha,k}^{p,q}$ space.

Proposition 1. (See [1], [4]-[6].)

If $0 < p_0 < p_1 < \infty, 0 < q \leq \infty, \delta_0, \delta'_0 > 0$ and $\frac{n+\delta'_0}{p_1} = \frac{n+\delta_0}{p_0}$ then

$$A_{\delta_0,k}^{p_0,q}(D) \subset A_{\delta'_0,k}^{p_1,q}(D).$$

Corollary 1. (See [1], [4]-[6].)

If $0 < p \leq 1, \alpha > -1$ then

$$A_{\alpha}^p(D) \subset A_{\beta}^1(D), \beta = \frac{n+1+\alpha}{p} - (n+1)$$

that is

$$\int_D |F(z)| \rho(z)^{\frac{n+1+\alpha}{p} - (n+1)} dv(z) \leq C_4 \|F\|_{A_{\alpha}^p}, F \in H(D).$$

Proposition 2. (See [1], [4]-[6].)

If $0 < p_0 < \infty, 0 < q_0 < q_1 \leq \infty, \delta_0 > 0$ and $k = 0, 1, 2$, then $A_{\delta,k}^{p_0,q_0}(D) \subset A_{\delta,k}^{p_1,q_1}(D)$.

Lemma 2.

Let $\delta > 0, t > t_0, t_0$ is large enough then, r_0 is fixed

$$r^{\delta} \cdot \left(\int_0^{r_0} \frac{R^{t-\delta} dR}{(r+R)^{t+1}} \right) \leq C_5.$$

Lemma 3. (See [1].)

Let $0 < p < 1, s > -1, r > 0, t = p(s+n+1) - (n+1)$. Then we have (even if we replace $|\tilde{\Phi}|^r$ by K , where K is a kernel of $r >$ type)

$$\left| \int_D |f(\xi)| \cdot |\tilde{\Phi}(z, \xi)|^r \cdot d(\xi)^s dv(\xi) \right|^p \leq C_6 \int_D |f(\xi)|^p \cdot |\tilde{\Phi}(z, \xi)|^{rp} \cdot d(\xi)^t dv(\xi).$$

Remark 1. This Lemma is valid if we replace α by ρ since $d(\xi) \asymp \rho(\xi)$. Below we denote this function also by $\delta(\xi)$.

Theorem 1.

Let $0 < p, q \leq 1, \alpha > \tilde{\alpha}_0$, $\tilde{\alpha}_0$ is large enough then

$$(T_\alpha f)(w) = \int_{\Lambda} |K_{\alpha_0}(z, w)| \cdot |f(z)| \cdot dv_\alpha(z), \alpha_0 = \alpha + n + 1$$

maps $A_\beta^{p,q}$ into $A_\beta^{p,q}$; for all $\beta, \beta > 0$, $dv_\alpha = \delta^\alpha dv$.

Remark 2. This theorem in the unit disk is a well known classical result about Bergman projection in the unit disk.

Proof of theorem 1. By lemma 3 above

$$|(T_\alpha f)(w)|^p \leq C_7 \int_{\Lambda} |K_{\alpha_0}(z, w)|^p (\delta(z))^{(n+1)(p-1)+(\alpha)p} \cdot |f(z)|^p \cdot dv(z).$$

Then by lemma 1 we have

$$\int_{\partial\Lambda_{\tilde{r}}} |(T_\alpha f)(w)|^p d\sigma_{\tilde{r}} \leq C_8 \int_0^{r_0} \int_{d\Lambda_r} (\delta(z))^{\alpha p + (n+1)(p-1)} \cdot |f(z)|^p \cdot \frac{1}{(r + \delta(z))^{(\alpha+n+1)p-(n)}} \cdot w(z) d\sigma_\varepsilon dr,$$

$w(z) \in C^\infty(\Lambda)$.

Let $q \leq p$. Then we have that using lemmas above and the fact that $\int_{\partial\Lambda_r} |f|^p$ is monotone

$$\begin{aligned} & \int_0^{\tilde{r}_0} \tilde{r}^{q/p\beta} \left(\int_{\partial\Lambda_{\tilde{r}}} |(T_\alpha f)(w)|^p d\sigma_{\tilde{r}} \right)^{q/p} \times \left(\frac{d\tilde{r}}{\tilde{r}} \right) \leq \\ & \leq C_9 \int_0^{\tilde{r}_0} (\tilde{r}^{\beta q/p-1}) \times \\ & \times \left(\int_0^{r_0} \int_{d\Lambda_r} (\delta(z))^{\alpha p + (n+1)(p-1)} \cdot |f(z)|^p \cdot \frac{1}{(r + \delta(z))^{(\alpha+n+1)p-(n)}} \cdot d\sigma_\varepsilon dr \right)^{q/p} \cdot d\tilde{r} \leq \\ & \leq C_{10} \int_0^{\tilde{r}_0} (\tilde{r}^{\alpha q + (n+1)q - nq/p-1}) \times \\ & \times \left(\int_{d\Lambda_r} |f(z)|^p d\sigma \right)^{q/p} \cdot \left(\int_0^{\tilde{r}_0} \frac{\tilde{r}^{\beta q/p-1} \cdot d\tilde{r}}{(r + \tilde{r})^{(\alpha+n+1)q - nq/p}} \right) \cdot dr \leq C_{11} \|f\|_{A_\beta^{p,q}}. \end{aligned}$$

Let $\frac{q}{p} > 1$, $\chi_\gamma(z) = \frac{1}{(1-|z|)^{\frac{\gamma}{pq}}}$, $z \in U = \{|z| < 1\}$, $T = \{|z| = 1\}$, $0 < \gamma < \gamma_0$.

We show our theorem in the unit disk since it is typical and repetition of our arguments leads to the proof in bounded pseudoconvex domains. We have repeating arguments above

$$\int_0^1 (1-\rho)^\beta \left(\int_T |T_\alpha(f)(\rho z)|^p \cdot d\xi dr \right)^{q/p} \rho d\rho \leq \\ \leq C_{12} \int_0^1 (1-\rho)^\beta \left(\int_0^1 \frac{(1-r)^{\alpha p+2(p-1)}}{(1-r\rho)^{(\alpha+2)p-1}} \int_T |f(r\xi)|^p d\xi dr \right)^{q/p} \cdot \rho d\rho = J(f).$$

Using Holder’s inequality and changing the order of integral we have that

$$J(f) \leq C_{13} \int_0^1 \left(\int_0^1 \frac{(1-r)^{\alpha p+2(p-1)}}{(1-r\rho)^{(\alpha+2)p-1} \cdot (\chi_\gamma^{q/p}(r))} \right) \cdot \left(\int_T |f(r\xi)|^p d\xi \right)^{q/p} \\ \cdot \left(\int_0^1 \frac{(1-r)^{\alpha p+2(p-1)} \cdot \chi_\gamma^{\frac{q}{q-p}}(r)}{(1-r\rho)^{(\alpha+2)p-1}} dr \right)^{\left(\frac{q}{q-p}\right)^{-1} \cdot (q/p)} (1-\rho)^\beta \leq \\ \leq C_{14} \int_0^1 (1-\rho)^\beta \cdot \chi_\gamma^{q/p}(\rho) \cdot \left(\int_0^1 \frac{(1-r)^{\alpha p+2(p-1)} r dr}{(1-r\rho)^{(\alpha+2)p-1} \chi_\gamma^{q/p}(r)} \right) \times \left(\int_T |f(r\xi)|^p d\xi \right)^{q/p} dr \rho d\rho \leq \\ \leq C_{15} \int_0^1 \frac{(1-r)^{\alpha p+2(p-1)}}{\chi_\gamma^{q/p}(r)} \cdot \left(\int_T |f(r\xi)|^p d\xi \right)^{q/p} \left(\int_0^1 \frac{(1-\rho)^\beta \chi_\gamma^{q/p}(\rho) d\rho}{(1-r\rho)^{(\alpha+2)p-1}} \right) dr \leq \\ \leq C_{16} \int_0^1 \frac{(1-r)^{\alpha p+2(p-1)} \times (1-r)^\beta (\chi_\gamma^{q/p}(r))}{(\chi_\gamma^{q/p}(r))(1-r)^{(\alpha+2)p-2}} \cdot \left(\int_T |f(r\xi)|^p d\xi \right)^{q/p} dr \leq C_2 \|f\|_{A_\beta^{p,q}}.$$

Theorem is proved.

Remark 3. We suppose that our theorem is valid for $A^{p,q}$ spaces when both $p, q \geq 1$ and even when p or q is infinite.

We define mixed norm analytic function spaces in bounded strongly pseudoconvex domains with boundary (in product domains).

Let

$$A_{\alpha_1, \alpha_2, \dots, \alpha_n}^{p_1, p_2, \dots, p_n}(D^m) = \\ = \left\{ f \in H(D \times \dots \times D) : \left(\int_D \dots \left(\int_D |f(z_1, \dots, z_m)|^{p_1} dv_{\alpha_1}(z_1) \right)^{p_2/p_1} \dots dv_{\alpha_m}(z_m) \right)^{1/p_m} < \infty \right\}$$

Changing A by L we denote the larger space consisting of all measurable functions in $D \times \dots \times D$, where $0 < p_i < \infty, -1 < \alpha_i < \infty, i = 1, \dots, m$.

We can obviously easily define similarly these spaces when one p_j index is infinite.

Theorem 2.

Let $p_j > 1, i = 1, \dots, m, \alpha_j > -1, j = 1, \dots, m$. Then we have that the V_{β} operator, where

$$\left(V_{\beta}\right)(f) = \int_D \dots \int_D f(z_1, \dots, z_m) \prod_{j=1}^m K_{\beta_j+n+1}(z_j, w_j) dv_{\beta_1}(z_1) \dots dv_{\beta_m}(z_m);$$

for all $\beta_j > \beta_0, j = 1, \dots, m; \beta_0$ is large enough maps $L_{\alpha_1, \dots, \alpha_n}^{p_1, \dots, p_n}$ into $A_{\alpha_1, \dots, \alpha_n}^{p_1, \dots, p_n}$ space.

Remark 4. In the unit disk for $m = 1$ this result is classical and well-known fact (Bergman projection theorem). The proof uses only Forelly-Rudin estimate from Lemma 1 and Holders and Minkowski inequalities and $m = 2$ and unit disk case is typical. We have in the unit disk $U = \{|z| < 1\}, m = 2$ case the following estimates.

We denote as usual by dm_2 the normalized Lebegues measure in the unit disk U .

Put first

$$D_{\alpha_j}(\xi_j, z_j) = \frac{\alpha_j + 1}{\pi} \cdot \frac{(1 - |\xi_j|)^{\alpha_j}}{(1 - \bar{\xi}_j z_j)^{\alpha_j+2}}, j = 1, 2;$$

$$D_{\alpha}(z, \xi) = D_{\alpha_1}(z_1, \xi_1) \times D_{\alpha_2}(z_2, \xi_2);$$

$$\frac{1}{p_j} + \frac{1}{q_j} = 1, \quad j = 1, 2, \quad \chi_{\gamma}(z_1, z_2) = (1 - |z_1|)^{\frac{-\gamma}{p_1 q_1}} \cdot (1 - |z_2|)^{\frac{-\gamma}{p_2 q_2}}, \quad z_j \in U, \quad \xi_j \in U, \quad j = 1, 2.$$

$$\begin{aligned} \|V_{\beta} f(\cdot, z_2)\|_{A_{\alpha_1}^{p_1}} &= \left(\int_U |F(z_1, z_2)|^{p_1} (1 - |z_1|)^{\alpha_1} dm_2(z_1) \right)^{1/p_1} \leq \\ &\leq C_{17} \left(\int_U \left(\int_U |D_{\alpha_1}(z_1, \xi_1)| \cdot |D_{\alpha_2}(z_2, \xi_2)| \cdot |f(\xi_1, \xi_2)| dm_2(\xi_1) dm_2(\xi_2) \right)^{p_1} (1 - |z_1|)^{\alpha_1} dm_2(z_1) \right) \leq \\ &\leq C_{18} \int_U |D_{\alpha_2}(z_2, \xi_2)| \left(\int_U \left(\int_U |D_{\alpha_1}(z_1, \xi_1)| \cdot |f(\xi_1, \xi_2)| dm_2(\xi_1) \right)^{p_1} (1 - |z_1|)^{\alpha_1} dm_2(z_1) \right)^{1/p_1} dm_2(\xi_2) \leq \\ &\leq C_{19} \int_U |D_{\alpha_2}(z_2, \xi_2)| \left(\int_U \left(\int_U \frac{|D_{\alpha_1}(z_1, \xi_1)| \cdot |f(\xi_1, \xi_2)|^{p_1} dm_1(\xi_1)}{|\chi_{\gamma}^{p_1}(\xi_1, \xi_2)|} \right) \times \right. \\ &\times \left. \left(\int_U |D_{\alpha_1}(z_1, \xi_1)| \cdot \chi_{\gamma}^{p_1}(\xi_1, \xi_2) dm_2(\xi_1) \right)^{p_1/q_1} (1 - |z_1|)^{\alpha_1} dm_2(z_1) \right)^{1/p_1} dm_2(\xi_2) \leq \end{aligned}$$

$$\begin{aligned} &\leq C_{20} \int_U |D_{\alpha_2}(z_2, \xi_2)| \left(\int_U \frac{|f(\xi_1, \xi_2)|^p}{\chi_\gamma^{p_1}(\xi_1, \xi_2)} \int_U |D_{\alpha_1}(z_1, \xi_1)| \chi_\gamma^{p_1}(z_1, \xi_2) (1 - |z_1|)^{\alpha_1} dm_2(z_1) dm_2(\xi_2) \right)^{1/p_1} dm_2(\xi_2) \leq \\ &\leq C_{21} \int_U |D_{\alpha_2}(z_2, \xi_2)| \cdot \|f(\cdot, \xi_2)\|_{L_{\alpha_1}^{p_1}} dm_2(\xi_2). \end{aligned}$$

Then we have that

$$\|V_{\vec{\beta}} f(\cdot, z_2)\|_{A_{\alpha_1, \alpha_2}^{p_1, p_2}}^{p_2} \leq C_{22} \int_U \left(\int_U |D_{\alpha_2}(z_1, \xi_2)| \cdot \|f(\cdot, \xi_2)\|_{L_{\alpha_1}^{p_1}} dm_2(\xi_2) \right)^{p_2} (1 - |z_2|)^{\alpha_2} dm_2(z_2).$$

Using again Holders inequality with p_2 and Forelly-Rudin estimate and changing the order of integration we get what we need

$$\|V_{\vec{\beta}}(f)\|_{A_{\alpha_1, \alpha_2}^{p_1, p_2}}^{p_2} \leq C \|f\|_{L_{\alpha_1, \alpha_2}^{p_1, p_2}}^{p_2}.$$

We finish the proof of our theorem using simple induction.

Theorem is proved.

Remark 5. We suppose that Similar results are valid when one index is infinite In our mixed norm A^{p_1, \dots, p_n} spaces.

Our projection theorems can be used in the study of action of Hankel and Toeplitz operators in bounded strongly pseudoconvex domains (see for example [7]).

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Об авторах:

Шамоян Роми Файзович, кандидат физико-математических наук, доцент кафедры «Высшая математика» Брянского государственного технического университета (241035, Россия, г. Брянск, ул. Харьковская, 10-Б), ORCID: <https://orcid.org/0000-0002-8415-9822>, rsham@mail.ru

Томашевская Елена Брониславовна, кандидат физико-математических наук, доцент кафедры «Высшая математика» Брянского государственного технического университета (241035, Россия, г. Брянск, ул. Харьковская, 10-Б), <https://orcid.org/0000.0002.1314.0550>, tomele@mail.ru

About the authors:

Romi Fayzovich Shamoyn, Cand.Sci. (Phys. & Math.), Associate Professor, Higher Mathematics Department, Bryansk State Technical University (241035, Russia, Bryansk, 10-B Kharkovskaya St.), ORCID: <https://orcid.org/0000-0002-8415-9822>, rsham@mail.ru

Elena Bronislavovna Tomashevskaya, Cand.Sci. (Phys. & Math.), Associate Professor of Higher Mathematics Department, Bryansk State Technical University (241035, Bryansk, Russia, 10-B Kharkovskaya St.), <https://orcid.org/0000-0002-1314-0550>, tomele@mail.ru

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